happen'd about the Time of the natural Birth, the Child then muſt have continued alive ſome conſiderable Time afterwards, during which theſe bony Excreſcences were formed; there being a perfect Oſſification, as performed by the Laws of Circulation, and not by any vegetative or petrifying Power, as in inanimate Bodies.

Two or three of the lateral Proceſſes of the Spine were what firſt paſſed thro' the little Ulcer; the reſt of the Bones (except a few that were loſt in cleaning) were preſented by the Doctor to the Muſeum of the *Royal Society*. They retain a very ſtrong and ſingular Smell, though they were immediately cleanſed from the rotten Fleſh, and well waſhed.

The Woman came by Sea to *Stockholm* above a Year after this Cure, and was preſented to the *Academy* in good Health; and the Doctor believes ſhe is ſtill alive and well.

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## II. *The Motion of* Projectiles *near the Earth's Surface conſider'd, independent of the Properties of the Conic Sections; in a Letter to* Martin Folkes *Eſquire, Pr. R. S. by Mr.* Tho. Simpſon *F. R. S.*.

*Read* Feb. 4. 1747.

AFTER ſo much as has been already ſaid upon the Motion of Projectiles *in vacuo*, it may ſeem needleſs to attempt any thing further on that Head; nevertheleſs, as a thorough Knowledge in the Art of Gunnery is become more than ever neceſſary, and as Gentlemen employ'd in the Practice of that Art are (I am ſenſible) too often deterr'd from applying themſelves to the Theory, by the Difficulties they imagine they ſhall meet with in the Conic Sections, you will, I hope, pardon the Liberty I have taken, in troubling you with my Thoughts on a Subject, in which little or nothing new is to be expected beſides the Method.

When I firſt drew up this Paper (which was about two Years ago) I did intend, had Health permitted me to make the proper Experiments, to have alſo attempted ſomething with reſpect to the Reſiſtance of the Atmoſphere, whereof the Effects are indeed too conſiderable to be intirely diſregarded: But if the Amplitude of the Projection, anſwering to one given Elevation, be firſt determined by Experiment (which our Method ſuppoſes) the Amplitudes in all other Caſes, where the Elevations and Velocities do not very much differ from the firſt, may be determined, by the Proportions here laid down, to a ſufficient Degree of Exactneſs: Becauſe, in all ſuch Caſes, the Effects of the Reſiſtance will be nearly as the Amplitudes themſelves; and were they accurately ſo, the Proportions of the Amplitudes, at different Elevations, would be exactly the ſame as *in vacuo*; which Proportions I now proceed to determine.

## PROBLEM I.

*Let two Balls be projected with the ſame Celerity, at different, but given Elevations, 'tis propoſed to determine the Ratio of the Times of their Flight,*

### *Flight, of thoir greateſt Altitudes, and of their horizontal Amplitudes.*

Let $Pq$ (*Fig.* 1.) repreſent the Plane of the Horizon, $PEQ$ and $peq$ the Paths of the Projectiles, deſcribed in the Flight; moreover let $QPT$ and $qpt$ be the given Angles of Elevation, and let $PQ$ and $pq$ be biſected in $H$ and $h$; drawing $HE$, $QT$, $he$ and $qt$, all perpendicular to $Pq$; and making the Sine of $QPT = S$, its Co-ſine $= C$, the Sine $qpt = s$, its Co-ſine $= c$, and Radius $= r$.

Therefore, ſince the Diſtances deſcended by heavy Bodies (whether from a Point at Reſt, or from the right Lines in which they *would* move, if not acted upon by Gravity) are known to be as the Squares of the Times, $QT$ will be to $qt$, as the Square of the Time of deſcribing $PEQ$ (or of that wherein the Ball would move uniformly over the Space $PT$ with its firſt Velocity at $P$) is to the Square of the Time of deſcribing $peq$ (or of that wherein the other Ball would move uniformly thro' the Length $pt$). But the Celerities at $P$ and $p$ being equal, by Hypotheſis, the Times in which the ſaid Lines $PT$ and $pt$ would be uniformly deſcribed, are manifeſtly, as the Lines themſelves: Whence the Squares of thoſe Lines muſt, alſo, be as the Squares of the Times, and, conſequently, as the Diſtances deſcended: that is, $Pt^2 : pt^2 :: TQ : tq$.

Now, by Plane Trigonometry $TQ = \frac{S \times PT}{r}$ and $tq = \frac{s \times pt}{r}$; therefore $PT^2 : pt^2$ $\left(:: \frac{S \times PT}{r} : \frac{s \times pt}{r}\right) :: S \times PT : s \times pt$; whence, by dividing the Antecedents by $PT$, and the Conſequents by $pt$, we have $PT \,.\, pt :: S : s$; from which it appears, that the Times of Flight are directly as the Sines of Elevation.

Again, the Times of deſcribing $EQ$ and $eq$ (which are the Halves of the Wholes) being alſo to one another as $S : s$, and the Diſtances $EH$, $eh$ deſcended in them, as the Squares of the Times, it likewiſe follows, that $S^2 : s^2 :: EH : eh$; or that the greateſt Altitudes are as the Squares of the Sines of Elevation.

Moreover, becauſe (by Trigonometry) $PT = \frac{r \times PQ}{C}$ and $pt = \frac{r \times pq}{c}$; and it has been already proved, that, $S : s :: PT : pt$, it follows, that $S : s :: \frac{r \times PQ}{C} : \frac{r \times pq}{c}$; whence, by multiplying the Antecedents by $\frac{2C}{r}$ and the Conſequents by $\frac{2c}{r}$, it will be $\frac{2SC}{r} : \frac{2cs}{r}$ $(:: 2PQ : 2pq) :: PQ : pq$. But $\frac{2SC}{r}$ is known to be the Sine of double the Angle whoſe Sine is $S$, and Co-ſine $C$, &c. Therefore the horizontal Amplitudes are to one another, as the Sines of the double Elevations.

### *Corol.* 1.

Hence it follows, that the greateſt Amplitude poſſible will be, when the Elevation is half a Right Angle, or 45 Degrees (becauſe the Sine of 90° is the greateſt of all others).

### *Corol.* 2.

Therefore, if the greateſt Amplitude be given (from Experiment) the Amplitude anſwering to any propoſed Elevation, above, or below, 45 Degrees, may from hence be found: For it will be as the Radius, to the Sine of double the given Elevation, ſo is the greateſt, to the required, Amplitude.

### *Corol.* 3.

Hence, alſo, the Altitude of the Projection may be known; for $QT$, when the Angle $QPT$ is half a Right Angle, will be $= PQ$; and therefore $HE$ $(\frac{1}{4}TQ) = \frac{1}{4}PQ$; alſo, in this Caſe, $S^2 = \frac{1}{2}r^2$; whence our Proportion $S^2 : s^2 :: HE : he$ will here become $\frac{1}{2}r^2 : s^2 :: \frac{1}{4}PQ : he$; from whence it appears, that, as the Square of the Radius is to the Square of the Sine of any given Elevation, ſo is half the greateſt horizontal Amplitude, to the Altitude of the Projection. Hence it alſo follows, that the Height to which the Ball would aſcend, if projected directly upwards, is juſt half the greateſt Amplitude.

### *Corol.* 4.

Therefore, ſince it is well known, that a Body *in vacuo* aſcends and deſcends with the ſame Velocity; and that the Diſtances deſcended are as the Squares of the Velocities; it follows, that the Amplitudes, at the ſame Elevation, with different Velocities, will alſo be to one another as the Squares of the Velocities; becauſe they are as the greateſt Amplitudes, with the ſame Velocities (by *Corol.* 2.)

and theſe are as the Diſtances perpendicularly deſcended (*by the precedent*). Whence, *univerſally*, if both the Elevations and the Velocities differ, the Amplitudes will be to each other in a *Ratio* compounded of the *Ratio's* of the Sines of double the Angles of Elevation, and of the duplicate *Ratio's* of the Velocities, or impelling Forces.

### *Problem* II.

*The Angle of Elevation, and the greateſt horizontal Amplitude, being given, to find at what Diſtance the Piece ought to be planted, to hit an Object, whoſe Diſtance, above or below the Plane of the Horizon, is alſo given.*

Let $AB$ (*Fig.* 2 and 3.) be the Plane of the Horizon, $BC$ the perpendicular Height or Depreſſion of the Object, and $AB$ the required Diſtance: Alſo let $BC$ be produced to meet the Line of Direction $AD$ in $D$, and let $P$ be the Place where the Path of the Projectile would meet the Horizon; moreover, let $PQ$ be perpendicular to $AP$, and $CN$ parallel to $AD$. Then, by the preceding Problem, it will be as Radius : the Sine of $2BAD$ :: the given (or greateſt) Amplitude : $AP$; which therefore, is known.

Moreover, the Areas of ſimilar Triangles being as the Squares of their homologous Sides, we have $AP\times PQ : AB\times BD :: AQ^2 : AD^2$. But $AQ^2 : AD^2 :: AB\times BD :: QP : DC$ (from Principles already explained) therefore, by Equality, $AP\times PQ : AB\times BD :: QP : DC$; and conſequently $AP : AB :: BD : CD$; but (becauſe of the parallel Lines $CN$ and $AD$) $BD : CD :: AB : AN$; whence, again by Equality, $AP : AB :: AB : AN$; therefore, by Diviſion, $AP : BP :: AB : BN$; and, conſequently $AP \times BN = BP \times AB$.

Let $AP$ be now biſected in $O$; then $BP \times AB$ being $= AO^2 - OB^2$ (in the firſt Caſe) and $= OB^2 - AO^2$ (in the ſecond Caſe), we ſhall therefore have $OB^2 = AO^2 \mp AP \times BN = AO \times \overline{AO \mp 2BN}$: whence the Diſtance $AB$ is likewiſe known. *Q.E.I.*

### *Corollary.*

Hence, if the Elevation, and the greateſt Amplitude, together with the Diſtance $AB$ of the Object be given, the Height or Depreſſion of the Ball in the Perpendicular $BCD$ will be known: For it is proved, that $AP : BP :: BA : BN$; whence $BN$ is known: But, as the Radius to the Tangent of $BNC$ ($BAD$): ſo is $BN$ to $BC$.

## *Problem* III.

*The greateſt horizontal Amplitudes of the Piece, together with the Diſtance and Height (or Depreſſion) of the Object being given, to find the Direction or Angle of Elevation.*

Let $BC$ (*Fig.* 4 and 5.) be the perpendicular Height or Depreſſion of the Object, $AB$ its given horizontal Diſtance, and $AH$ the required Direction; Alſo let $PQ$ (*Fig.* 6.) be the greateſt Amplitude (anſwering to 45° of Elevation); draw $AC$, in which produced (if need be) take $AG = PQ$; make $MGO$ perpendicular to $AG$, meeting $AB$ produced (if need be) in $O$; and from the Centre $O$, with the Interval $OA$, let a Circle be deſcribed, interſecting $AG$, produced in $E$, and the Line of Direction $AD$ in $H$; join $E$, $H$, and let $HI$, $AN$ and $QR$, be perpendicular to $AE$, $AO$, and $PQ$ reſpectively, and let $BC$, produced, meet $AH$ in $D$.

It will appear, from what has been ſaid above, that $AD^2 : PR^2 :: DC : RQ$; therefore $PR^2$ being $= 2PQ^2 = 2AG^2 = \frac{1}{2}AE^2$, and $RQ = PQ = \frac{1}{2}AE$ (by Conſtruction), we have $AD^2 : \frac{1}{2}AE :: DC : \frac{1}{2}AE$, and therefore $AD^2 = AE \times DC$.

Now, the Triangles $ADC$, $AEH$, being equiangular (becauſe $ADC = DAN = AEH$, and $DAC$ common to both) we likewiſe have $AD : DC :: AE : EH$, and conſequently $AE \times DC = AD \times EH = AD^2$ (*per* above); whence $EH = AD$. Therefore, as the Triangles $ADB$ and $EHI$ are equiangular, they are equal in all reſpects; and ſo $HI = AB$: Whence follows this eaſy

## *Conſtruction.*

Having deſcribed the Circle $AEF$, as above directed, and drawn $MG$ perpendicular to $AE$, take $Gn$ equal to $AB$, and thro' $n$, parallel to $AE$, draw $Hh$, cutting the Circle in $H$ and $h$; join $A$, $H$, and $A$, $h$; then either of the Directions $AH$ or $Ah$, will anſwer the Conditions of the Problem. From this Conſtruction we have the following Calculation, *viz.*

As $AB$ is to $BC$, ſo is $AG$ to $OG$; which added to, or ſubtracted from, $Gn$ ($AB$) gives $On$: Then, it will be, as $AG : On ::$ the Co-ſine of $OAG$ : Co-ſine of $HOn$ ($= HAh$) the Difference of the two required Elevations; whence the Elevations themſelves are known. *Q. E. I.*

### *Corol.* 1.

Hence, if the Elevation of the Piece, with the Diſtance and the Height (or Depreſſion) of the Object be given, the greateſt horizontal Amplitude may be found: For it will be $AB : BC ::$ Radius : Tang. of $BAC$; whence $CAD$ is alſo known.

Then, $S. CAD : S. ACD$ ($AHE$) $:: AD$ ($HE$) $: AE$.

And, $S. ADC :$ Radius $:: AB : AD$.

Therefore, by compounding theſe Proportions, we have $S. CAD \times S. ADC :$ Radius $\times S. ACD :: AB : AE$; which is equal to twice the required Amplitude, by Conſtruction.

### *Corol.* 2.

Moreover, if the Elevation, and the greateſt horizontal Amplitude be given, the Amplitude of the Projection on any aſcending or deſcending Plane $AE$, whoſe Inclination $FAE$ is alſo given, may from hence be derived. For, $S. AHE$ ($ACD$) $: S : EAH$ ($CAD$) $:: AE$ ($2PQ$) $: EH$ ($AD$) and $S. ACD : S. ADC :: AD : AC$; whence, by compounding the two Proportions, $Sq^r. S. ACD : S. CAD \times S. ADC :: 2PQ : AC$; from which $AC$ is known.

### *Corol.* 3.

Since it appears, that the Triangles $ADB$ and $EHI$ are equal and alike in all reſpects, and, therefore, the horizontal Diſtance $AB$, *univerſally*, equal to the Perpendicular $HI$, it is manifeſt, that, when

$HI$ is the greateſt poſſible, $AB$ will alſo be the greateſt poſſible; in which Circumſtance $AC$ (if the Angle $FAE$ be given) will likewiſe be the greateſt poſſible: And this, it is evident, muſt be, when $HI$ coincides with $MG$, or when the Angles $HEA$ and $HAE$ are equal (as in *Fig.* 7 and 8); at which time the Point $D$ coincides with $H$; becauſe $AD$ and $EH$ are always equal to each other. Therefore, ſince, in this Caſe, $HAE$ ($HEA$) is $= NAH$, it follows, that the Amplitude, on any inclined Plane $AE$, will be the greateſt poſſible, when the Line of Direction $AH$ biſects the Angle made by the Plane and Zenith.

*Corol.* 4.

Hence the greateſt Amplitude on any inclined Plane may alſo be known; for the right-angled Triangles $AOG$ and $HOB$, having $AO = HO$ and the Angle $O$ common, are equal in all reſpects; and therefore, as Tang. of $AHG$ ($BAH$ the Angle of Elevation): Tang. of $CHG$ ($CAB$ the Plane's Inclination) $:: AG : GC$; whence $AC = AG \mp GC$ is alſo known.

*Corol.* 5.

Hence, alſo, if the greateſt Amplitude on an inclin'd Plane be given, the greateſt horizontal Amplitude may be determined: For, Radius $: S.\,BAC ::$ $AC : BC = CG =$ the Difference of the given, and the required, Amplitudes.

*Corol.* 6.

But if, inſtead of the Plane's Inclination, the perpendicular Height, or Depreſſion, of the Object be given; then, $AC$ ($AG \mp BC$) being to $BC$, as Radius to the Sine of $BAC$, and Radius : Cotang. $BAC$ :: $BC : AB$; the greateſt Diſtance $AB$, at which the Ball can poſſibly hit the Object, will from hence be given: which Diſtance (becauſe $AC = AG \mp BC$, and $AB^2 = \overline{AC + CB} \times \overline{AC - BC}$) will alſo be expreſſed by $\sqrt{AG \times \overline{AG \mp 2BC}}$. Hence the greateſt horizontal Amplitude of a Ball, projected from a given Height above the Plane of the Horizon is known: For $ST$ (*Fig.* 8.) may here be ſuppoſed to repreſent the Plane of the Horizon, and $SA$ the given Height; and then $SC$, being equal to $AB$, is given from above $= \sqrt{AG \times \overline{AG + 2BC}}$.

### *Corol.* 7.

But, if the horizontal Diſtance $AB$ be given, and it be required to find the greateſt Height the Ball can poſſibly reach in the Perpendicular $BCD$; we ſhall have $HG$ ($AB$) : $AG$ :: Radius : Tang. of the Elevation ($BAH$ or $AHG$); and Radius : Tang. $BAC$ ($2BAH \backsim 90^\circ$) :: $AB : BC$; which therefore is known. But (becauſe $AC \pm BC = AG$, and $\overline{AC + CB} \times \overline{AC - CB} = AB^2$) the ſame will alſo be truly exhibited by $\frac{AG^2 \backsim AB^2}{2AG}$.

### *Corol.* 8.

Laſtly, let the Height, or Depreſſion, of the Object be given, together with its Diſtance $AB$, to determine the Direction, and the leaſt *Impetus* poſſible, to hit the Object: Then $AB : BC$ :: Radius : Tang. $BAC$; whence the Elevation $BAH$ is known: And as Radius : Tang. $AHG$ ($BAH$) :: $MG$ ($AB$) : $AG$; whence the *Impetus* is alſo known.

*Philos. Trans. N.° 486.*

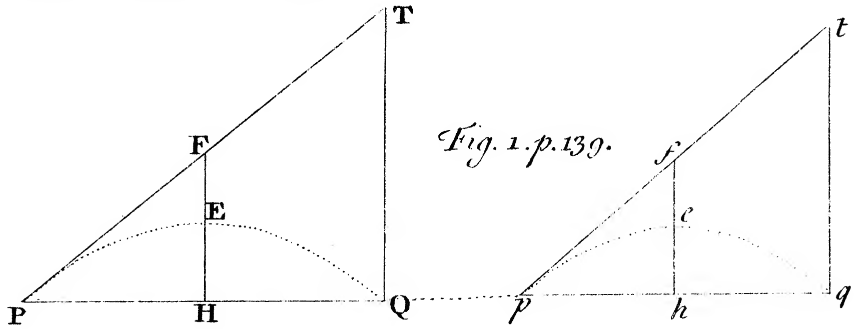


*Fig. 1. p. 139.*

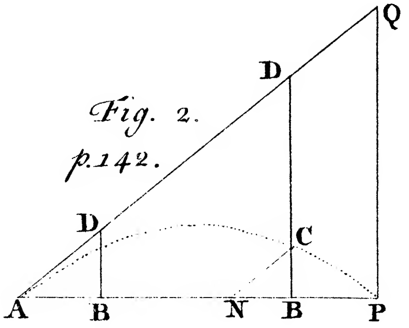


*Fig. 2. p. 142.*

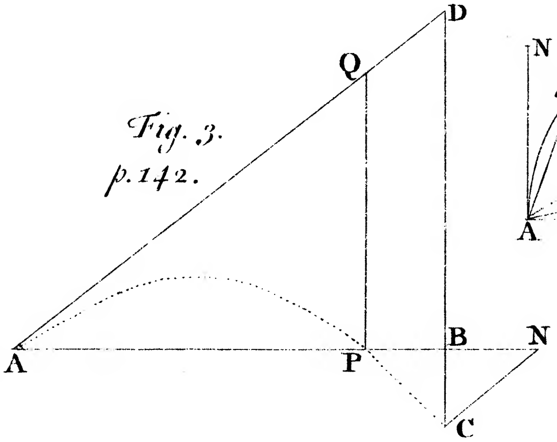


*Fig. 3. p. 142.*

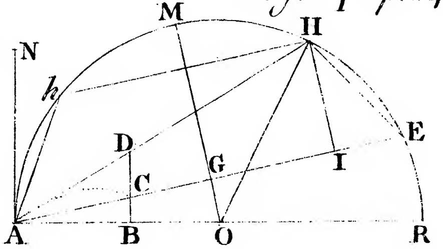


*Fig. 4. p. 143.*

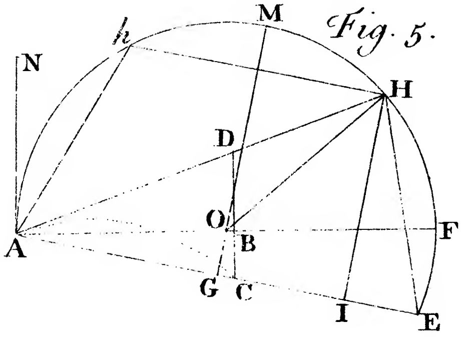


*Fig. 5.*

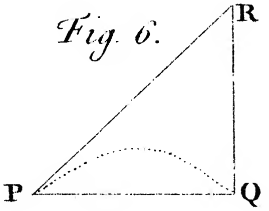


*Fig. 6.*

*p. 146.*

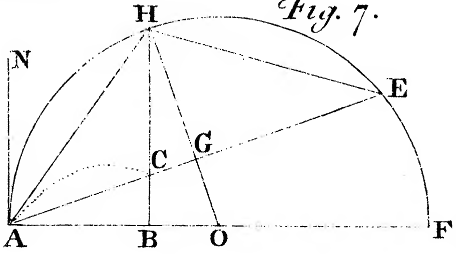
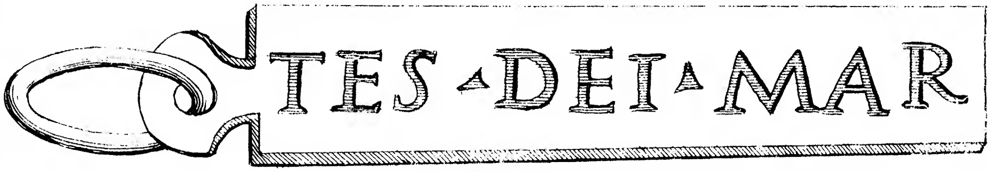
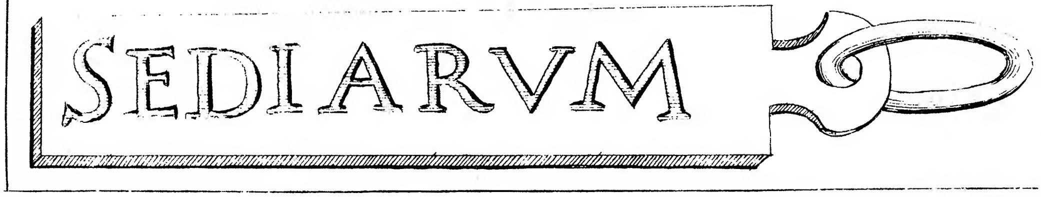


*Fig. 7.*



*p. 225.*



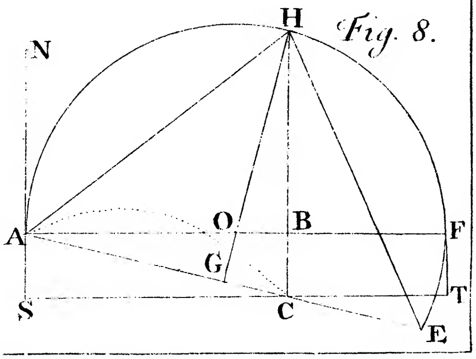


*Fig. 8.*